Pergamon
www.elsevier.com/locate/jappmathmech
PII: S0021-8928(02)00034-5
J. Appl. Maths Mechs, Vol. 66, No. 2, pp. 275–281, 2002

0021–8928/02/$—see front matter

THE METHOD OF FICTITIOUS ABSORPTION IN COUPLED MIXED PROBLEMS OF THE THEORY OF ELASTICITY AND MATHEMATICAL PHYSICS FOR A MULTILAYERED INHOMOGENEOUS HALF-SPACE†

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(*Received* 10 *July* 2001)

The method of fictitious absorption [1–3] is generalized to a class of coupled mixed problems of the theory of electroelasticity, thermoelasticity and mathematical physics for a multilayered inhomogeneous half-space. These problems differ in the fact that the symbols of the kernels of their integral equations, together with the zeros and poles, have branching points. The generalization is based on the use of direct numerical procedures, which enables a wide class of matrix functions of simple structure to be used, subject to only a single requirement, namely, the asymptotic properties of the symbols of the kernels of the corresponding integral operators must be preserved. In this case the form of representation of the symbol of the kernel in order to realize the numerical algorithms is determined by the type of medium considered. An approximate representation of the symbol of the kernel of the integral operator is used when it is a meromorphic function (a layer, a packet of layers, etc.). In this case, replacement of the symbol of an approximating function of simple structure enables the computational resources required to achieve the required accuracy of the solution to be reduced. The qualitative nature of the solution is preserved over the whole volume. An accurate representation of the symbol of the kernel of the integral operator is used when, together with the zeros and poles, it has branching points on the real axis (a multilayered inhomogeneous half-space) or in its neighbourhood (thermoelastic media). The use in this case of approximating functions is not very effective since it leads to errors in the solution not only of a quantitative nature but also of a qualitative nature, in view of the impossibility of constructing uniform approximations of the symbol of the kernel, which take into account the presence of branching points in it.

An investigation of coupled mixed problems of the theory of electroelasticity or thermoelasticity assumes the use of effective methods of solving systems of integral equations for a certain extended vector function, whose components, together with the components of the contact-stress vector, is either the distribution density of the charge (a component of the induction vector), or the thermal flux density in the contact region. These methods include the method of fictitious absorption ([1–3], etc.), developed to solve systems of equations which arise when investigating mixed problems of the theory of electroelasticity or thermoelasticity for media in the form of a layer or multilayered packages. The symbols of the kernels of such equations are meromorphic functions. However, the approach proposed previously ([1–3], etc.) is inapplicable to integral equations whose kernel symbols have branching points. Moreover, the application of this approach assumes the use of special functionally commutative matrices of complex structure. The approach proposed in this paper to solving systems of integral equations is completely free of these drawbacks.

1. SCHEME OF THE METHOD

Consider a system of integral equations

$$\mathbf{kq} = \iint\limits_{\Omega} \mathbf{k}(x_1 - \xi, x_2 - \eta)\mathbf{q}(\xi, \eta)d\xi d\eta = \mathbf{f}(x_1, x_2), \quad x_1, x_2 \in \Omega \tag{1.1}$$

$$\mathbf{k}(s,t) = \frac{1}{4\pi^2}\int\limits_{\Gamma_1}\int\limits_{\Gamma_2} \mathbf{K}(\alpha_1, \alpha_2)e^{-i(\alpha_1 s + \alpha_2 t)}d\alpha_1 d\alpha_2, \quad \mathbf{K}(\alpha_1, \alpha_2) = \| K_{mn}(\alpha_1, \alpha_2)\|_{m,n=1}^{N}$$

The dimension of the system is determined by the type of medium and the geometry of the problem. In three-dimensional problems for electroelastic media, ignoring thermal effects, as well as in problems of thermoelasticity when there are no electric fields, $N = 4$. For pyroelectric media $N = 5$, etc.

†*Prikl. Mat. Mekh.* Vol. 66, No. 2, pp. 285–292, 2002.

The elements $K_{mn}(\alpha_1, \alpha_2)$ possess the following properties [1]:

1) they are analytical functions of two complex variables, which allow of the representation

$$K_{mn}(\alpha_1,\alpha_2)=\sum_{\lambda=1}^{C_{mn}} P^{\lambda}_{mn}(\alpha_1,\alpha_2)R^{\lambda}_{mn}(u),\quad u=\sqrt{\alpha_1^2+\alpha_2^2},\quad C_{mn}<\infty$$

where $P^{\lambda}_{mn}(\alpha_1, \alpha_2)$ are certain polynomials and $R^{\lambda}_{mn}(u)$ are analytical functions, such that the following relation holds

$$\det \mathbf{K}(\alpha_1,\alpha_2)=D(u)$$

where $D(u)$ is an analytical function of the variable u;

2) the functions $R^{\lambda}_{mn}(u)$ and $D(u)$ can have a finite number of branching points and a finite number of real poles;

3) as $|\alpha_i| \to \infty$ in the system of coordinates $\alpha_1^{\gamma}O\alpha_2^{\gamma}(\alpha_1^{\gamma} = \alpha_1 \cos\gamma + \alpha_2 \sin\gamma,\ \alpha_2^{\gamma} = \alpha_2 \cos\gamma - \alpha_1 \sin\gamma)$, the following asymptotic representations hold

$$R^{\lambda}_{mn}(u)=(c^{\lambda}_{mn}\,|u|^{\theta(1,\lambda,m,n)}+d^{\lambda}_{mn}u^{\theta(2,\lambda,m,n)})(1+O(u^{-1})),\quad \theta(k,\lambda,m,n)<0$$

$$K^{\gamma}_{mn}(\alpha_1^{\gamma},\alpha_2^{\gamma})=|\alpha_2^{\gamma}|^{-1}\,[a_{mn}+b_{mn}\operatorname{sign}\alpha_2^{\gamma}][1+O((\alpha_2^{\gamma})^{-1})]$$

where

$$K^{\gamma}_{mn}(\alpha_1^{\gamma},\alpha_2^{\gamma})=K_{mn}(\alpha_1^{\gamma}\cos\gamma-\alpha_2^{\gamma}\sin\gamma,\alpha_1^{\gamma}\sin\gamma+\alpha_2^{\gamma}\cos\gamma),$$

and $a_{mn} = a_{mn}(\alpha_1^{\gamma}, \gamma)$ and $b_{mn} = b_{mn}(\alpha_1^{\gamma}, \gamma)$ are bounded functions of the parameters.

System (1.1) is uniquely solvable for any doubly continuous differentiable function $f(x_1, x_2)$ [1]. The region Ω whose boundary can have corner points, is convex. The location of the contours Γ_1 and Γ_2 ensures that the radiation conditions [4] are satisfied.

We will introduce a matrix S, the elements of which have a simple structure and possess asymptotic properties of the corresponding elements of the matrix K. It can be shown that the matrix

$$\mathbf{\Pi}(\alpha_1,\alpha_2)=\mathbf{S}^{-1}(\alpha_1,\alpha_2)\mathbf{K}(\alpha_1,\alpha_2) \tag{1.2}$$

which, when $|\alpha_1|$, $|\alpha_2| \to \infty$, has the representation

$$\mathbf{\Pi}(\alpha_1,\alpha_2)=\mathbf{I}+O(u^{-1})$$

where $\mathbf{I}$ is the identity matrix. The following matrix has a similar property

$$\boldsymbol{\pi}(\alpha_1,\alpha_2)=\mathbf{\Pi}^{-1}(\alpha_1,\alpha_2)=\mathbf{K}^{-1}(\alpha_1,\alpha_2)\,\mathbf{S}(\alpha_1,\alpha_2) \tag{1.3}$$

We construct the following matrices

$$\mathbf{\Pi}^{*}(\alpha_1,\alpha_2)=\|\Pi^{*}_{mn}\|_{m,n=1}^{N},\quad \boldsymbol{\pi}^{*}(\alpha_1,\alpha_2)=\|\pi^{*}_{mn}\|_{m,n=1}^{N}$$

whose elements are rational functions which approximate the elements of the matrices Π and π on the real axis

$$\Pi_{mn}\approx\Pi^{*}_{mn}=P^{0}_{mn}(\alpha_1,\alpha_2)P^{0^{-1}}(u),\quad \pi_{mn}\approx\pi^{*}_{mn}=\pi^{0}_{mn}(\alpha_1,\alpha_2)\pi^{0^{-1}}(u)$$

and we will introduce the following matrix

$$\mathbf{K}_0=\mathbf{K}\mathbf{\Pi}^{*^{-1}} \tag{1.4}$$

which, while possessing the asymptotic properties of the matrix $\mathbf{K}$, includes all features ignored in $\mathbf{\Pi}^*$ including branching points. We will further assume that the polar sets of the matrices $\mathbf{\Pi}$ and $\boldsymbol{\pi}$, as well as $\mathbf{\Pi}^*$ and $\boldsymbol{\pi}^*$, are defined by the relations

$$\alpha_1^2+\alpha_2^2=z_k^2\ \text{ and }\ \alpha_1^2+\alpha_2^2=\zeta_k^2,\quad k=1,2,\dots,M.$$

Lemma [1]. Suppose the vector function $\mathbf{q}(x_1, x_2) \in \mathbf{L}_p(\Omega)$, $p > 1$ has a carrier in the convex region Ω. In order that the vector function ($\mathbf{V}(\alpha_1, \alpha_2)$ and $\mathbf{V}^{-1}(x_1, x_2)$ are the direct and inverse Fourier transformation)

$$\mathbf{t}(x_1, x_2) = \mathbf{V}^{-1}(x_1, x_2)\mathbf{\Pi}(\alpha_1, \alpha_2)\mathbf{V}(\alpha_1, \alpha_2)\mathbf{q}$$

should possess this property, it is necessary and sufficient that the identity $\mathbf{V}(\alpha_1, \alpha_2)\mathbf{q} \equiv 0$ holds in the polar set of the function $\mathbf{\Pi}(\alpha_1, \alpha_2)$.

Following the approach developed earlier [-3], the solution of system (1.1) will be sought in the form

$$\mathbf{q}(x_1, x_2) = \mathbf{q}_0(x_1, x_2) + \mathbf{g}(x_1, x_2), \quad \mathbf{g} = \sum_{k=1}^{4M^2} \mathbf{C}_k g_k(x_1, x_2) \tag{1.5}$$

Here g_k is a set of certain functions, defined in the region Ω, and $C_k = \{C_k^i\}_{i=1}^N$ are vectors which satisfy the condition

$$\mathbf{V}(\alpha_1, \alpha_2)\mathbf{q} = \mathbf{V}(\alpha_1, \alpha_2)\mathbf{g}, \quad \mathbf{\Pi}^*\mathbf{V}(\alpha_1, \alpha_2)\mathbf{q}_0 = 0 \tag{1.6}$$
$$\alpha_1^2 + \alpha_2^2 = z_k^2, \quad k = 1, 2, \ldots, M$$

After substituting expressions (1.5) into system (1.1), taking Eq. (1.4) into account, we obtain, after some reduction, the system

$$\mathbf{k}_0\mathbf{t} = \int_{\Gamma_1}\int_{\Gamma_2} \mathbf{K}_0(\alpha_1, \alpha_2)\mathbf{T}(\alpha_1, \alpha_2)e^{-i(x_1\alpha_1 + x_2\alpha_2)}d\alpha_1 d\alpha_2 = \mathbf{f}(x_1, x_2) - \sum_{i=1}^{N}\sum_{k=1}^{4M^2} C_k^i \mathbf{f}_k^i \tag{1.7}$$

in the new unknown function

$$\mathbf{T}(\alpha_1, \alpha_2) = \mathbf{\Pi}^*(\alpha_1, \alpha_2)\mathbf{V}(\alpha_1, \alpha_2)\mathbf{q}_0 \tag{1.8}$$

Here

$$\mathbf{t} = \{t^n\}_{n=1}^N, \quad \mathbf{K}^0 = \| K_{nj}^0 \|_{n,j=1}^N, \quad \mathbf{f} = \{f^n\}_{n=1}^N, \quad \mathbf{f}_k^i = \{f_k^{in}\}_{n=1}^N$$

$$f_k^{in}(x_1, x_2) = \iint_\Omega k_{ni}(x_1 - \xi, x_2 - \eta)g_k(\xi, \eta)d\xi d\eta$$

It follows from relation (1.8) that

$$\mathbf{q}_0 = \mathbf{V}^{-1}\boldsymbol{\pi}^*\mathbf{V}\mathbf{t}, \quad \mathbf{t}(x_1, x_2) = \mathbf{V}^{-1}(x_1, x_2)\mathbf{T} \tag{1.9}$$

the same region as $\mathbf{t}$. Hence, the following relations must be satisfied in the polar set $\boldsymbol{\pi}^*(\alpha_1, \alpha_2)$

$$\mathbf{T}(\alpha_1, \alpha_2) = 0 \tag{1.10}$$

From relations (1.5), after introducing expressions (1.9), we obtain the final form of the Fourier transformant of the solution of system (1.1)

$$\mathbf{Q}(\alpha_1, \alpha_2) = \boldsymbol{\pi}^*(\alpha_1, \alpha_2)\mathbf{T}(\alpha_1, \alpha_2) + \mathbf{V}(\alpha_1, \alpha_2)\mathbf{g} \tag{1.11}$$

We obtain the solution of system (1.1) by applying to expression (1.11) the Fourier inversion

$$\mathbf{q}(x_1, x_2) = \mathbf{V}^{-1}\boldsymbol{\pi}^*\mathbf{V}\mathbf{t} + \mathbf{g} \tag{1.12}$$

Remark 1. When investigating the dynamics of massive bodies and mechanical and electromechanical systems which interact with thermoelastic or electroelastic media, it is sufficient to calculate the integral characteristics of the problem (the response of the medium to the action of a punch, the thermal flux through the contact region, the charge, etc.). There is no need to calculate the density of these characteristics (the contact stresses, the thermal flux density, the charge density distribution, etc.). In this case it is best to use formula (1.11), making the substitution

$$\alpha_1 = \alpha_2 = 0, \quad g_k = \delta(x_1 - x_1^k, x_2 - x_2^k)$$

Here x_1^k, x_2^k are the coordinates of the vertices of the rectangles dividing the region Ω.

2. Representation (1.12) holds for $\mathbf{g} \in \mathbf{L}_p(\Omega)$. When using a system of δ-functions in relations (1.5), it is best to introduce the operator [1]

$$\mathbf{sg} = \iint_\Omega \int_{\Gamma_1} \int_{\Gamma_2} \mathbf{K}_0(\alpha_1,\alpha_2)[\mathbf{\Pi}^*(\alpha_1,\alpha_2) - \mathbf{I}]e^{-i(\alpha_1(x_1-\xi)+\alpha_2(x_2-\eta))} d\alpha_1 d\alpha_2 \mathbf{g}(\xi,\eta) d\xi d\eta$$

Expression (1.12) then becomes

$$\mathbf{q}(x_1,x_2) = \mathbf{k}_0^{-1}\mathbf{f} - \mathbf{k}_0^{-1}\mathbf{sg} + \mathbf{V}^{-1}[\boldsymbol{\pi}^* - \mathbf{I}]\mathbf{V}[\mathbf{k}_0^{-1}\mathbf{f} - \mathbf{g} - \mathbf{k}_0^{-1}\mathbf{sg}]$$

2. SOLUTION OF THE SYSTEM

We will seek a solution of system (1.7) in the form

$$\mathbf{t}(x_1,x_2) = \mathbf{t}_0(x_1,x_2) - \sum_{i=1}^{N}\sum_{k=1}^{4M^2} C_k^i \mathbf{t}_k^i(x_1,x_2) \tag{2.1}$$

where $\mathbf{t}_0$ and $\mathbf{t}_k^i$ satisfy the equations

$$\mathbf{k}_0\mathbf{t} = \mathbf{f}(x_1,x_2), \quad \mathbf{k}_0\mathbf{t}_k^i = \mathbf{f}_k^i \tag{2.2}$$

We introduce two systems of basis (coordinate) functions $\psi_k(x_1, x_2)$ and $\varphi_k(x_1, x_2)$, specified in the region Ω and possessing completeness [5]. We will represent the approximate solutions of Eqs (2.2) in the form

$$\mathbf{t}_0(x_1,x_2) = \sum_{m=1}^{L} \boldsymbol{\beta}_m \psi_m(x_1,x_2), \quad \mathbf{t}_k^i(x_1,x_2) = \sum_{m=1}^{L} \boldsymbol{\beta}_{km}^i \psi_m(x_1,x_2) \tag{2.3}$$

where $\boldsymbol{\beta}_m = \{\beta_m^j\}_{j=1}^N$ and $\boldsymbol{\beta}_{km}^i = \{\beta_{km}^{ij}\}_{j=1}^N$ $(m = 1, 2, \dots, L)$ are vectors of dimensionality N.

Substituting the first expression of (2.3) into the first equation of (2.2) and using the Bubnov–Galerkin method and changing the order of summation, which is related to the transition from vectors of dimensionality N to vectors of dimensionality L, we obtain the matrix system

$$\sum_{j=1}^{N} \mathbf{A}^{nj}\boldsymbol{\beta}^j = \mathbf{F}^n, \quad n = 1,2,\dots,N \tag{2.4}$$

$$\mathbf{A}^{nj} = \| A_{lm}^{nj} \|_{l,m=1}^L, \quad A_{lm}^{nj} = \int_{\Gamma_1}\int_{\Gamma_2} K_{nj}^0(\alpha_1,\alpha_2)\Psi_m(\alpha_1,\alpha_2)\Phi_l^*(\alpha_1,\alpha_2) d\alpha_1 d\alpha_2$$

$$\boldsymbol{\beta}^j = \{\beta_m^j\}_{m=1}^L, \quad \mathbf{F}^n = \{F_l^n\}_{l=1}^L, \quad F_l^n = \iint_\Omega f^n(x_1,x_2)\varphi_l(x_1,x_2) dx_1 dx_2$$

Here K_{nj}^0 and f^n are the components of the matrix $\mathbf{K}_0$ and the vector $\mathbf{f}$, $\Psi_m(\alpha_1, \alpha_2)$ and $\Phi_l(\alpha_1, \alpha_2)$ are the Fourier transformations of the functions $\psi_m(x_1, x_2)$ and $\varphi_l(x_1, x_2)$, and the asterisk denotes a complex-conjugate quantity.

Acting in the same way with the second expression of (2.3), we arrive at the matrix system

$$\sum_{j=1}^{N} \mathbf{A}^{nj}\boldsymbol{\beta}_k^{ij} = \mathbf{F}_k^{in}, \quad n = 1,2,\dots,N, \quad k = 1,2,\dots,4M^2 \tag{2.5}$$

$$\boldsymbol{\beta}_k^{ij} = \{\beta_{km}^{ij}\}_{m=1}^L, \quad \mathbf{F}_k^{in} = \{F_{kl}^{in}\}_{l=1}^L, \quad F_{kl}^{in} = \iint_\Omega f_k^{in}(x_1,x_2)\varphi_l(x_1,x_2) dx_1 dx_2$$

Systems (2.4) and (2.5) are systems of equations of order N with matrix coefficients in N unknown vectors of dimensionality L. The comparatively low order of matrix systems (2.4) and (2.5), related to the dimensionality of the initial problem, enables us to separate the problem into two stages, solving these systems initially for the unknown vectors in analytical form and obtaining N algebraic systems of order L, and then using numerical methods to solve the latter.

The elements of the matrices A^{nj} are calculated once. The remaining actions in calculating the whole set of vectors $\boldsymbol{\beta}^i$ and $\boldsymbol{\beta}_k^{in}$ reduces to successive multiplication of these matrices and their combinations by the vectors $\mathbf{F}^i$ and $\mathbf{F}_k^{in}$. The use of this approach to solve a system of two equations with two unknowns will be described below.

We will assume that the solutions of systems (2.4) and (2.5) have been obtained and that the functions $\mathbf{t}_0$ and $\mathbf{t}_k^i$ have been constructed. Substituting expression (2.1) into (1.9) we obtain a system of $N \times 4M^2$ equations for obtaining the coefficients C_k^i.

$$\mathbf{T}_0(\alpha_1,\alpha_2)-\sum_{i=1}^{N}\sum_{k=1}^{4M^2} C_k^i \mathbf{T}_k^i(\alpha_1,\alpha_2)=0, \quad \alpha_1^2+\alpha_2^2=\zeta_k^2, \quad k=1,2,\ldots,M \tag{2.6}$$

3. EXAMPLE

As an example, we will consider a system of integral equations which are typical when investigating, in the two-dimensional formulation, coupled mixed problems of electroelasticity for piezoactive crystals of class 6 mm or piezoceramics,

$$\int_{-a}^{a} \mathbf{k}(x_1-\xi,\omega)\mathbf{q}(\xi)d\xi=\mathbf{f}(x_1), \quad |x_1|\leqslant a; \quad \mathbf{k}(s,\omega)=\frac{1}{2\pi}\int_{\Gamma} \mathbf{K}(\alpha)e^{i\alpha s}d\alpha \tag{3.1}$$

where $\mathbf{K}(\alpha)$ is a second-order matrix function whose elements are even functions having the same poles, and also branching points on the real axis; the following representations hold as $\alpha \to \infty$

$$K_{nn}(\alpha)=c_n|\alpha|^{-1}[1+O(\alpha^{-1})], \quad K_{12}(\alpha)=K_{21}(\alpha)=b|\alpha|^{-1}[1+O(\alpha^{-1})] \tag{3.2}$$

Bearing expressions (3.2) in mind, we introduce the following matrix

$$\mathbf{S}(\alpha)=(\alpha^2+B^2)^{-1/2}\left\|\begin{matrix} a_1 & b \\ b & a_2 \end{matrix}\right\|$$

the elements of which must only possess asymptotic properties of the corresponding elements of the symbol $\mathbf{K}(\alpha)$. Matrices (1.2) and (1.3) in this case take the form

$$\boldsymbol{\Pi}(\alpha)=\left\|\begin{matrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{matrix}\right\|, \quad \boldsymbol{\pi}(\alpha)=\left\|\begin{matrix} \Pi_{22} & -\Pi_{12} \\ -\Pi_{21} & \Pi_{11} \end{matrix}\right\| \tag{3.3}$$

$$\Pi_{nj}=\Pi_0 R_{nj}, \quad \pi_{nj}=\pi_0 r_{nj}, \quad r_{nn}=R_{3-n,3-n}, \quad r_{nj}=-R_{nj}, \quad n\neq j, \quad n,j=1,2$$

$$R_{1n}(\alpha)=c_2K_{1n}-bK_{2n}, \quad R_{2n}(\alpha)=c_1K_{2n}-bK_{1n}, \quad n=1,2$$

$$\Pi_0=\frac{\sqrt{\alpha^2+B^2}}{\Delta}, \quad \pi_0=\frac{1}{\Pi_0\Delta D(\alpha)}, \quad D(\alpha)=K_{11}K_{22}-K_{12}^2, \quad \Delta=c_1c_2-b^2$$

By construction $\Pi_{nj}(\alpha)$ are even functions, $\Pi_{nn}(\alpha)\to 1+O(\alpha^{-1})$, $\Pi_{nj}(\alpha)\to\alpha^{-2}[1+O(\alpha^{-1})]$ $(n\neq j)$ when $\alpha\to\pm\infty$; they have zeros $\pm\gamma_{knj}(k=1,2,\ldots,N_{nj})$ and also the same poles $\pm z_k(k=1,2,\ldots,M)$. The elements π_{nj} possess similar properties, and have zeros $\pm\beta_{knj}(k=1,2,\ldots,N_{nj})$, where $\beta_{k11}=\gamma_{k22}$, $\beta_{k22}=\gamma_{k11}$, $\beta_{k12}=\gamma_{k12}$, $\beta_{k21}=\gamma_{k21}$, and also the same poles $\pm\zeta_k(k=1,2,\ldots,M)$. In both cases $M\geqslant N_{nj}$.

We will approximate $\Pi_{nj}(\alpha)$ and $\pi_{nj}(\alpha)$ by the rational functions

$$\Pi^*_{nj}(\alpha)=\prod_{k=1}^{N_{nj}}(\alpha^2-\gamma^2_{knj})\prod_{k=1}^{M}(\alpha^2-z_k^2)^{-1} \tag{3.4}$$

$$\pi^*_{nj}(\alpha)=\prod_{k=1}^{N_{nj}}(\alpha^2-\beta^2_{knj})\prod_{k=1}^{M}(\alpha^2-\zeta_k^2)^{-1},\quad N_{nj}=\begin{cases}M, & n=j\\ M-1, & n\neq j\end{cases}$$

We will seek a solution of Eq. (3.1) in the form

$$\mathbf{q}(x_1)=\mathbf{q}_0(x_1)+\mathbf{g}(x_1),\quad \mathbf{g}=\sum_{k=1}^{2M}\mathbf{C}_k\delta(x_1-x_1^k) \tag{3.5}$$

where x_1^k are the coordinates of points which divide the segment $[-a, a]$ into equal parts. It can be seen that the function $\mathbf{g}$ satisfies relations (1.6).

After reduction, involving substituting expressions (3.5) into system (3.1) and a subsequent change in the order of summation, we arrive at the following system

$$\mathbf{k}_0\mathbf{t}=\mathbf{f}(x_1)-\sum_{k=1}^{2M}[C_k^1\mathbf{f}_k^1+C_k^2\mathbf{f}_k^2] \tag{3.6}$$

$$\mathbf{f}_k^m=\{f_k^{m1},f_k^{m2}\},\quad f_k^{mn}=k_{nm}(x_1-x_1^k)$$

in the new unknown function

$$\mathbf{t}(x_1)=\mathbf{V}^{-1}(x_1)\mathbf{\Pi}^*(\alpha)\mathbf{V}(\alpha)\mathbf{q}_0 \tag{3.7}$$

By virtue of the linearity, we will seek a solution of system (3.6) in the form

$$\mathbf{t}(x_1)=\mathbf{t}_0(x_1)-\sum_{k=1}^{2M}[C_k^1\mathbf{t}_k^1(x_1)+C_k^2\mathbf{t}_k^2(x_1)] \tag{3.8}$$

where $\mathbf{t}_0$ and $\mathbf{t}_i^k$ satisfy the systems

$$\mathbf{k}_0\mathbf{t}=\mathbf{f}(x_1),\quad \mathbf{k}_0\mathbf{t}^i=\mathbf{f}_k^i \tag{3.9}$$

We will introduce two systems of basis functions $\psi_m(x_1)$ and $\varphi_m(x_1)$ ($m = 1, 2, \ldots, L$), possessing completeness and defined in the segment $[-a, a]$. We will represent the approximate solution of systems (3.9) in the form

$$\mathbf{t}_0(x_1)=\sum_{m=1}^{L}\boldsymbol{\beta}_m\psi_m(x_1),\quad \mathbf{t}_k^i(x_1)=\sum_{m=1}^{L}\boldsymbol{\beta}_{km}^i\psi_m(x_1) \tag{3.10}$$

where $\boldsymbol{\beta}_m = \{\beta_m^1, \beta_m^2\}$ and $\boldsymbol{\beta}_{km}^i = \{\beta_{km}^{i1}, \beta_{km}^{i2}\}$ are unknown coefficients.

Substituting the first expression of (3.10) into the first equation of (3.9) and using the Bubnov–Galerkin method, after reduction with respect to the change in the order of summation, we arrive at the system

$$\mathbf{A}^{n1}\boldsymbol{\beta}^1+\mathbf{A}^{n2}\boldsymbol{\beta}^2=\mathbf{F}^n,\quad n=1,2 \tag{3.11}$$

$$\mathbf{A}^{nj}=\| A_{lm}^{nj}\|_{l,m=1}^{L},\quad A_{ml}^{nj}=\int_\Gamma K_{nj}^0(\alpha)\Psi_m(\alpha)\Phi_l^*(\alpha)d\alpha$$

$$\boldsymbol{\beta}^j=\{\beta_m^j\}_{m=1}^{L},\quad \mathbf{F}^n=\{F_l^n\}_{l=1}^{L},\quad F_l^n=\int_{-\infty}^{\infty}f^n(x_1)\psi_l(x_1)dx_1,\quad (n=1,2)$$

Here $\Psi_m(\alpha)$ and $\Phi_l(\alpha)$ are Fourier transformants of the basis functions.

We will represent the solution of system (3.11) in the matrix form

$$\boldsymbol{\beta}^1 = \mathbf{A}^{11^{-1}}\mathbf{F}^1 - \mathbf{A}^{11^{-1}}\mathbf{A}^{12}\boldsymbol{\beta}^2, \quad \boldsymbol{\beta}^2 = \mathbf{B}\cdot[\mathbf{F}^2 - \mathbf{A}^{21}\mathbf{A}^{11^{-1}}\mathbf{F}^1] \tag{3.12}$$

$$\mathbf{B} = [\mathbf{A}^{22} - \mathbf{A}^{21}\mathbf{A}^{11^{-1}}\mathbf{A}^{12}]^{-1}$$

Acting in the same way with the second expression of (3.10), we obtain the system

$$\mathbf{A}^{n1}\boldsymbol{\beta}_k^{i1} + \mathbf{A}^{n2}\boldsymbol{\beta}_k^{i2} = \mathbf{F}_k^{in}, \quad n = 1,2 \tag{3.13}$$

$$\boldsymbol{\beta}_k^{in} = \{\beta_{km}^{in}\}_{m=1}^{L}, \quad \mathbf{F}_k^{in} = \{F_{kl}^{in}\}_{l=1}^{L}, \quad F_{kl}^{in} = \int_{-\infty}^{\infty} f_k^{in}(x_1)\psi_l(x_1)dx_1$$

which is also a system of equations with matrix coefficients and which differs from (3.11) only in the right-hand sides. Its solution has the form

$$\boldsymbol{\beta}_k^{i1} = \mathbf{A}^{11^{-1}}\mathbf{F}_k^{i1} - \mathbf{A}^{11^{-1}}\mathbf{A}^{12}\boldsymbol{\beta}_k^{i2}, \quad \boldsymbol{\beta}_k^{i2} = \mathbf{B}\cdot[\mathbf{F}_k^{i2} - \mathbf{A}^{21}\mathbf{A}^{11^{-1}}\mathbf{F}_k^{i1}] \tag{3.14}$$

Hence, the fundamental problem of solving system (3.7) has been reduced to constructing the matrices $A_{ij}(i, j = 1, 2, \ldots, N)$, and to calculating $\mathbf{A}_{11}^{-1}$ and $\mathbf{B}$. The remaining actions to determine the whole set of vectors $\boldsymbol{\beta}^i$ and $\boldsymbol{\beta}_k^{in}$ reduces to successive multiplication of these matrices by the vectors $\mathbf{F}^i$ and $\mathbf{F}_k^{in}$ and their combinations.

The Fourier transformant of the solution of Eq. (3.1) has the form

$$\mathbf{Q}(\alpha) = \boldsymbol{\pi}^*(\alpha)\mathbf{T}(\alpha) + \mathbf{V}(\alpha)\mathbf{g} \tag{3.15}$$

The coefficients C_k^i are found from the relations

$$\mathbf{T}_0(\alpha) - \sum_{i=1}^{2}\sum_{k=1}^{2M} C_k^i\mathbf{T}_k^i(\alpha) = 0, \quad \alpha_1 = \pm\zeta_k, \quad k = 1,2,\ldots,M \tag{3.16}$$

Applying an inverse Fourier transformation to expression (2.15) we obtain

$$\mathbf{q}(x_1) = \mathbf{V}^{-1}\boldsymbol{\pi}^*\mathbf{V}\mathbf{t} + \mathbf{g}$$

This research was supported financially by the Russian Foundation for Basic Research (99-01-00787, 99-01-01015, 00-01-9602n r2000yug), the Federal Special-Purpose "Integration" Programme (A0017), and the US Civilian Research and Development Foundation (REC-004-A). The opinion of the Foundation may not coincide with the conclusions of the paper.

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Translated by R.C.G.